

Undetermined Coefficients Method

Ex 1: Solve the i.v.p. using the Undetermined Coefficients Method

$$\vec{Y}' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \vec{Y} + \underbrace{\begin{bmatrix} e^t \\ 0 \end{bmatrix}}_{G(t)}, \quad \vec{Y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\textcircled{1} \Phi = \begin{bmatrix} e^{3t} & e^{-t} \\ e^{2t} & -e^{-t} \end{bmatrix}$$

$$G = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$

$$\text{Guess } Y_p = e^t a = e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \Rightarrow a_1, a_2 = ?$$

② sub Y_p into $Y' = PY + G$

$$\underbrace{\left(e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right)'}_{e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \right) + \begin{bmatrix} e^t \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} a_1 + 2a_2 + 1 \\ 2a_1 + a_2 + 0 \end{bmatrix}$$

$$\begin{cases} a_1 = a_1 + 2a_2 + 1 & \Rightarrow a_2 = -1/2 \\ a_2 = 2a_1 + a_2 & a_1 = 0 \end{cases}$$

$$Y_p = e^t \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = e^t \begin{bmatrix} 0 \\ -1/2 \end{bmatrix}$$

$$Y = Y_c + Y_p = \Phi \cdot C + \begin{bmatrix} 0 \\ -1/2 e^t \end{bmatrix}$$

